2050C Special Tutorial 2

We review 3.2, 3.3, 3.4, 3.5.

$\boxed{3.2}$ You should know

- Limit theorem
- Squeeze theorem
- $\lim_{n\to\infty} a_n = a \Rightarrow \lim_{n\to\infty} |a_n| = |a|$

  $\lim_{n\to\infty} a_n = a$, $a_n \geq 0$, $\Rightarrow \lim_{n\to\infty} \sqrt{a_n} = \sqrt{a}$.
- Ratio Test, $a_n \neq 0$,

  $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = L, \quad L \in (-1, 1) \Rightarrow \lim_{n\to\infty} a_n = 0$$

$\boxed{3.3}$

- Monotone Convergence Theorem
- Two examples: $e_n = (1+\frac{1}{n})^n$ increasing, bdd above, $e = \lim_{n\to\infty} e_n$.

  $S_{n+1} = \frac{1}{2}(S_n + \frac{a}{S_n})$, $S_1 > 0$, $a > 0$
  decreasing, bdd below, $\sqrt{a} = \lim_{n\to\infty} S_n$.

$\boxed{3.4}$

- Subsequences
- Divergence Criterion: $\{a_n\}$ diverges if either ⓐ $\exists$ unbdd subsequence or ⓑ $\exists$ two convergent subsequences with diffeent limit.
- Bolzano-Weierstrass Theorem.

$\boxed{3.5}$

- Cauchy sequence
- Cauchy Convergence Criterion: $\{a_n\}$ is convergent iff it is

a Cauchy sequence. (you should understand the proof well.)

- An example : $x_n = f_n / f_{n+1}$ on Pg 89, already appeared in midterm.

Ex. 2

1. Use ε-n_ε definition and Limit theorem to find

$$\lim_{n\to\infty} \frac{n^2-6n+1}{3n^2-5n}$$

2. Evaluate

$$\lim_{n\to\infty} \sqrt{(n+5)(n+10)} - n$$

3. Evaluate

$$\lim_{n\to\infty} \frac{a^n}{n!} \quad (a>0)$$

4. $$\lim_{n\to\infty} \frac{a^{n+1}+b^{n+1}}{a^n+b^n} \quad (0<a<b)$$

5. Show divergence

(a) $\{-1, 1, -1, 1, \cdots\}$

(b) $\{1, \frac{1}{2}, 2, \frac{1}{3}, 3, \frac{1}{4}, \cdots\}$

(c) $\{\cos \frac{n}{2}\pi\}_{n=1}^{\infty}$

6. Prove Cauchy Convergence Criterion.